(Serge Burckel 2007)

SECOND NEIGHBOURHOOD PROBLEM

1. Generalization to EVERY directed graphs

CONJ1 : Every non empty and finite directed graph G has a vertex X such that $\#GG_2(X) \geq \#GG_1(X)$ where :

$GG_1(X)=$
$\{Y : (X\neq Y) \text{ and } (X\rightarrow Y) \text{ and NOT } (Y\rightarrow X) \}$

$GG_2(X)=$
$\{Z : (X\neq Z) \text{and NOT } (X\rightarrow Z) \text{ and } \exists Y\ (X\neq Y)\text{and}(X\rightarrow Y)\text{and}(Y\rightarrow Z)\}$

2. Generalization to larger cycles

CONJ2 : For every $K \geq 2$, every non empty and finite directed graph G with no circuit of length 1,2,…,K has a vertex X such that $\#G_K(X) \geq \#G_{K-1}(X)$ where : $G_D(X) = \{Y \text{ at distance } D \text{ from } X\}$

3. A question that implies the SNP :

CONJ3 : Every directed graph G with no 1,2-circuit and with a unique solution X to the SNP relation $\#G_2(X) \geq \#G_1(X)$ satisfies :
"X has no out-neighbours", ie, $G_1(X)$ is empty.

4. A stronger question that implies the SNP :

CONJ4 : The algorithm GO terminates from every digraph.

(see my preprints on SNP)